DIAGNOSTIC AND PEDAGOGICAL ISSUES WITH MATHEMATICAL WORD PROBLEMS

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The Counting On program was designed to support the professional learning of teachers in identifying and addressing the learning needs of students in the middle years experiencing difficulties with the early mathematical concepts and skills. The program has undergone major changes such as the inclusion of Newman's Error Analysis (NEA) in 2007. The inclusion of NEA was the result of an unusual journey, beginning in the state of Victoria Australia, travelling to the country of Brunei Darussalam before returning to the state of New South Wales Australia. Newman (1977, 1983) defined five specific reading skills crucial to performance on mathematical word problems. In this paper data will be examined arising from the Counting On evaluations focussing on student outcomes and how teachers have used NEA as a remedial and general classroom pedagogical strategy.

The Counting on Numeracy Program

The Counting On program conducted by the New South Wales Department of Education and Training (NSWDET) was implemented in 1999 to address the needs of students who are excluded from effective mathematics study in the middle years and beyond because of a lack of understanding of and proficiency with the early school mathematical knowledge.

The Counting On program has a twin learning focus upon student and teacher learning and has continued to expand and evolve. The initial program was designed for secondary school students (Year 7) who had not achieved specific New South Wales Stage 3 mathematics outcomes by the time they commenced secondary school. It was later extended to include the primary schools and the middle years (9-14 year olds).

The research base for the program was provided through the Counting On Numeracy Framework (Thomas, 1999) which was an extension of work by Cobb and Wheatley (1988), Beishuizen (1993), Jones, Thornton, Putt, Hill, Mogill, Rich and van Zoest (1996) and relates to the Count Me In Too Learning Framework in Number, LFIN, (Wright, 1998; Wright, Martland, & Stafford, 2000).

This theoretical base was supported by an increasing research base provided by the regular Counting On evaluation studies (Mulligan, 1999, Perry & Howard, 2000, 2002a, 2003; White 2008, 2009). In 2007 the program underwent a major revision and was

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implemented in 122 schools across the state grouped into 30 clusters with each cluster supported by a mathematics consultant. It was based on the previous models but included changes designed to simplify and encourage further and ongoing involvement of schools. Features of the revised model included: a simplified assessment instrument; the inclusion of Newman's Error Analysis (NEA); a revised Counting On CD; formation of School clusters; a facilitator's conference; and a facilitated professional development model. It is the inclusion of NEA to help teachers diagnose and remedy student difficulties with word problems that is the focus of this paper.

**Newman's Error Analysis**

The place and importance of mathematical word problems in the school curriculum have attracted diverse opinions. "Teachers seem not to like word problems. Many have asked me why these are used to 'trick' children in assessments" (Askew, 2003, p. 78). It is well recognised that students appear to struggle with both the literacy and mathematical demands of typical mathematical word problems.

... at the upper primary level most errors on mathematics tests and examinations are caused by Reading, Comprehension or Transformation errors, or by Carelessness. Often, pupils are able to carry out one or more of the four operations (+, -, x, ÷) needed to answer a question, but they do not know which operations to use (Clements, 2004, p. ii).

The reasons for the inclusion of Newman’s Error Analysis (NEA, Newman, 1977; 1983) in the 2007 and 2008 programs were to assist teachers when confronted with students who experienced difficulties with mathematical word problems. While most mathematical questions involve the use of words, not all are classed as word problems. A primary condition of word problems is the inclusion of a word description of a context within which the problem resides.

Teachers who were confronted with students experiencing difficulties with word problems now had another strategy. Rather than giving students ‘more of the same’ involving increased drill and practice and hoping that students would overcome whatever difficulty they were having, NEA provided a framework for considering the reasons that underlay the difficulties and a process that assisted teachers to determine where misunderstandings occurred and where to target effective teaching strategies to overcome them. Moreover, it provided an opportunity for presenting excellent professional learning for teachers and made a nice link between literacy and numeracy.

NEA was designed as a simple diagnostic procedure. Newman (1977, 1983) maintained that when a person attempted to answer a standard, written, mathematics word problem then that person had to be able to pass over a number of successive hurdles: Level 1 Reading (or Decoding), 2 Comprehension, 3 Transformation, 4 Process Skills, and 5 Encoding (see Table 1 for the interview prompts). Along the way, it was always possible to make a careless error and there were some who gave incorrect answers because they were not motivated to answer to their level of ability. Newman's research generated a large amount of evidence highlighting that far more children experienced difficulty with the semantic structures, the vocabulary, and the symbolism of mathematics than with the standard algorithms. In many Newman studies carried out in schools the proportion of errors first occurring at the Comprehension and Transformation' stages has been large (Marinas &
Clements, 1990; Ellerton & Clements, 1996; Singhatat, 1991). Thus, studies regularly reported that approximately 70 per cent of errors made by Year 7 students on typical mathematics questions were at the Comprehension or Transformation levels. These researchers also found that Reading (Decoding) errors accounted for less than 5 per cent of initial errors and the same was true for Process Skills errors, mostly associated with standard numerical operations (Ellerton & Clarkson, 1996). Also, Newman's research consistently pointed to the inappropriateness of many remedial mathematics programs in schools in which the revision of standard algorithms was overemphasised, while hardly any attention was given to difficulties associated with Comprehension and Transformation (Ellerton & Clarkson, 1996).

Table 1

<table>
<thead>
<tr>
<th>The Newman's Error Analysis Interview Prompts</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Please read the question to me. If you don't know a word, leave it out.</td>
</tr>
<tr>
<td>2. Tell me what the question is asking you to do.</td>
</tr>
<tr>
<td>3. Tell me how you are going to find the answer.</td>
</tr>
<tr>
<td>4. Show me what to do to get the answer. &quot;Talk aloud&quot; as you do it, so that I can understand how you are thinking.</td>
</tr>
<tr>
<td>5. Now write down your answer to the question.</td>
</tr>
</tbody>
</table>

In Australia, NEA was mainly promoted by Clements (1980, 1982, 1984) and in collaboration with Ellerton (e.g., Clements & Ellerton, 1992, 1993, 1995; Ellerton, & Clements, 1991, 1996, 1997) during the 1980s and 1990s although there were others (e.g., Casey, 1978; Clarkson, 1980; Watson, 1980; Tuck, 1983; Faulkner, 1992). NEA was also widely used throughout the Asia-Pacific region such as in Brunei (Mohidin, 1991); in India (Kaushil, Sajjin Singh & Clements, 1985); in Malaysia (Marinas & Clements, 1990; Ellerton & Clements, 1992; Sulaiman & Remorin, 1993); in Papua New Guinea (Clements, 1982; Clarkson, 1983, 1991); in Singapore (Kaur, 1995); in the Philippines (Jiminez, 1992); and in Thailand (Singhatat, 1991; Thongtawat, 1992). This initial momentum declined and its inclusion in the Counting On program in 2007 was via an unusual path. Clements became a professor at the Universiti Brunei Darussalam and was heavily involved in a national professional learning program for primary teachers titled, Active Mathematics In Classrooms (AMIC; White & Clements, 2005). NEA was one aspect of this program. The AMIC program was reported in the primary journal, Square One, for the Mathematics Association of New South Wales. An article on NEA from Square One (White, 2005) was selected and added to the teacher reader section of the NSWDET's website in 2006 which created a renewed interest by teachers. In 2007 it was added to the Counting On program.

There have been adaptations to NEA and two will now be briefly described. The first is Casey (1978) who modified the interview procedures used by Newman (1977). In a study of the errors made by 120 Grade 7 students in a single high school, the interviewers were required to help students over errors. If a pupil made a Comprehension error, the interviewer would note this and explain the meaning of the question to the pupil, and so on. So, in Casey's study, a pupil could make a number of errors on the one question and thus it is difficult to compare Casey's interpretations with Newman's. Some teachers prefer this adaptation when their purpose is teaching and learning rather than diagnosis.

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The second adaption was proposed by Ellerton and Clements (1997) who used a modified form of the Newman interview method to analyse the responses by students in Grades 5 through 8 to a set of 46 questions. All responses, both correct and incorrect, were analysed. A correct answer which, after analysis, was not deemed to be associated with an adequate understanding of the main concepts, and/or skills and/or relationships tested by a question, would be associated with a Newman error category, even though the answer was correct. Ellerton and Clements' modification led to the adoption of a slightly different definition of "Careless" error from that previously given by Clements.

While there are other theoretical approaches available to teachers, NEA offers one of the easiest to use and adapt and has proven popular among teachers for the ease of the diagnostic features. What is also surprising is how NEA has been used by teachers as a problem solving strategy for students and a pedagogical strategy.

The next sections will briefly describe the methodology adopted and the data will be reported under the headings of student learning and teacher learning.

Methodology

The 2008 program was implemented in 99 schools across the NSW state. An assessment instrument of six questions, based on the LFIN, was administered by the classroom teacher to the class and covered place value, addition, subtraction, multiplication, division, and word problem tasks. These class results were then used to group the students as expert (correct working and answers to 5 or 6 items and clear understanding of correct number concepts needed to solve the problems), intermediate (some correct working and answers and some understanding of number concepts needed to solve the problems but still not fully developed or consistent) and target (few or no correct working or answers and evidence of misconceptions in working and answers). The target group was then interviewed and their levels recorded. They were retested on the same test and interviewed at the finish of the program. It is argued that repetition of the test would not influence the results as the students received no feedback on the initial test and there was at least a 10 week gap between assessments. The facilitators were asked to record the results of the target group assessment process involving a minimum of five students per class on an Excel spreadsheet supplied to them. The spreadsheet recorded the initial level on the LFIN and NEA scales for the targeted students before the program was implemented and again following 10 weeks of targeted activities. These results were compiled and are reported in the next section.

The use of a testing procedure raises the issue of whether a correct answer equates to understanding. Ellerton and Olson (2005) conducted a study of 83 Grades 7 and 8 North American students completing a test comprising items from Illinois Standards Achievement Tests. Their findings reinforced the fact that students' scores on tests do not necessarily reflect their level of understanding of mathematical concepts and relationships. Results indicated a 35% mismatch with students who gave correct answers with little or no understanding and others who gave incorrect answers but possessed some understanding. While these findings cast doubt on the use of large sample testing programs as a means of making comparisons or being used as basis for the allocation of resources, it may be less of an issue for the Counting On program as the groups of targeted students are small for each school and teachers make use of instruments LFIN and NEA which are designed to assist teachers in diagnosing the level of student understanding.
In 2008 data was collected from 74 schools with 55 primary schools, 16 secondary schools and 3 central schools. There were 1213 students with 954 primary students (78.6%) and 259 secondary students (21.4%). Only one of the two questions involving Newman’s Error Analysis in the assessment instrument was recorded for each student. The question used is presented in figure 1 above. The NEA scale from 1 to 5 was used, and a category 6 was added to represent those who could complete the word problem successfully.

Natalie paddled 402 km of the Murray River in her canoe over 6 days. She paddled the same distance each day. How far did Natalie paddle each day?

*Figure 1. A typical word problem.*

The Counting On program is funded under an Australian federal government program and there is a mandatory evaluation process that includes instruments and reporting requirements. The Counting On program also has to report to other NSW state bodies and other data is collected for these purposes. The author of this paper was given all the data collected from all the instruments used and asked to analyse and construct an evaluation report. He had neither input into the design of these instruments nor the collection of data although he was able to collect further data. Thus there are methodological issues that arise such as a concern with the initial and final student level diagnosis by teachers. While the facilitators are trained in the use of NEA, there are concerns with the process involving the other teachers and this will be discussed later in this paper. However as a result of these concerns with the integrity of some of the data, it was decided to use only simple statistical tools in the analysis.

In the next section, data from the 2007 and 2008 evaluation reports (White, 2008, 2009) will examine the student learning outcomes and the teacher uses of NEA. NEA was not measured directly in the 2007 report.

**Results**

**Student Learning**

Table 2 below displays the initial (pre-test) and final (post-test) NEA levels and indicates an improvement in the overall levels from the initial to the final student assessments.

<table>
<thead>
<tr>
<th>NEA Levels</th>
<th>Initial Level Frequency</th>
<th>Percentage Frequency</th>
<th>Final Level Frequency</th>
<th>Percentage Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>196</td>
<td>16.2%</td>
<td>51</td>
<td>4.2%</td>
</tr>
<tr>
<td>2</td>
<td>452</td>
<td>37.3%</td>
<td>234</td>
<td>19.3%</td>
</tr>
<tr>
<td>3</td>
<td>399</td>
<td>32.9%</td>
<td>477</td>
<td>39.3%</td>
</tr>
<tr>
<td>4</td>
<td>101</td>
<td>8.3%</td>
<td>220</td>
<td>18.1%</td>
</tr>
<tr>
<td>5</td>
<td>37</td>
<td>3.1%</td>
<td>134</td>
<td>11.0%</td>
</tr>
<tr>
<td>6</td>
<td>28</td>
<td>2.3%</td>
<td>97</td>
<td>8.0%</td>
</tr>
<tr>
<td>Total</td>
<td>1213</td>
<td>100.0%</td>
<td>1213</td>
<td>100.0%</td>
</tr>
</tbody>
</table>
Table 3 below shows that the majority of students have improved by 1 or more levels (56.6%), with a sizeable group improving two levels (15.6%). There are a small group of students who improved by 3 and 4 levels as there are some who decline by 1, 2 or more levels.

Table 3  
*The Difference In Newman's Error Analysis Levels for Counting On 2008*

<table>
<thead>
<tr>
<th>Difference</th>
<th>Frequency</th>
<th>Percentage Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>3</td>
<td>0.2%</td>
</tr>
<tr>
<td>-3</td>
<td>6</td>
<td>0.5%</td>
</tr>
<tr>
<td>-2</td>
<td>14</td>
<td>1.2%</td>
</tr>
<tr>
<td>-1</td>
<td>52</td>
<td>4.3%</td>
</tr>
<tr>
<td>0</td>
<td>452</td>
<td>37.3%</td>
</tr>
<tr>
<td>1</td>
<td>385</td>
<td>31.7%</td>
</tr>
<tr>
<td>2</td>
<td>189</td>
<td>15.6%</td>
</tr>
<tr>
<td>3</td>
<td>79</td>
<td>6.5%</td>
</tr>
<tr>
<td>4</td>
<td>27</td>
<td>2.2%</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>0.5%</td>
</tr>
<tr>
<td>Total</td>
<td>1213</td>
<td>100.0%</td>
</tr>
</tbody>
</table>

The descriptive statistics record an increase in the mean from 2.52 for the initial level (SD = 1.096) to 3.37 for the final level (SD = 1.254). There is a difficulty here in that these statistics rely on the assumption of the NEA levels being either a ratio or interval scale which is questionable regarding the equality of the distances between any two of the levels. Using a paired sample T-Test, the results indicate that the improvement in the student outcomes for mathematical word problem levels at the start and finish of the 10 week Counting On 2008 program was statistically significant.

The 2008 data collected for the pre- and post-program student learning outcomes indicated that a statistically significant improvement existed in student learning outcomes between the start of the program and the completion of the program in mathematical problem solving involving word problems. In a short program as this it is unrealistic to expect that all students will make great leaps up the NEA levels. These targeted students have been struggling for some time with their mathematical and literacy levels and have developed judgements of their own ability. To improve 1 level, especially for the NEA scale which could involve the improvement of reading or comprehension, is quite remarkable in such a small time frame.

There are possible explanations for lack of progress or regression of levels such as students who do not have the capacity to handle the mathematics required, or they have become very resistant due to negative experiences and poor self image. Vaiyatvutjamai and Clements (2004) analysed the errors made by 231 Form 3 (Year 9) Thai students in two Chiang Mai government secondary schools. Students completed tasks before and immediately after a series of 13 lessons. A number of misconceptions were revealed and although some were clarified as a result of the lessons, there were others that remained and seemed to be "fossilised". A "fossilised misconception" was used to denote the situation where a student maintains a faulty conception despite having been specifically taught the "official" defining characteristics of the relevant concept. Associated with this then is the absence of cognitive change over time or even resistance to change over time, so that
cognitive inertia persists despite the individual having been taught the "proper" view of the concept. The implications for this current study are that the strategies and procedures of the intervention program of Counting On should become integrated into the everyday classroom and continue after the program has finished. These "fossilised misconceptions" may require a greater time period for them to be changed.

Also of interest is that while the study by Vaiyatvutjamai and Clements (2004) involved students across the range of abilities, the results for low performing students challenged the use of the term misconception for many of the student errors.

A misconception can be regarded as a fairly stable, but inappropriate, way of thinking ... analysing the errors made by low performers in this study, was that the word "stable" was not one that could sensibly be used (p. 181).

Students with "unstable" conceptions will give different answers at different times and hence it is possible that their test scores will decline. Students who have not developed confidence in their ability to answer a question may revert to guessing. This may well explain the 35% mismatch with students who gave correct answers with little or no understanding and others who gave incorrect answers but possessed some understanding reported in Ellerton and Olson's (2005) study. While students using a guessing strategy may cause some instability in the results for the process level in NEA if not identified by the teacher, it would be easily revealed by the second adaption of NEA (Ellerton & Clements, 1997) where correct answers are interrogated.

This completes the section on NEA and student learning and the next section will discuss the second learning focus which is upon the teacher. Teachers were shown how to use the NEA prompts (see Table 1) to diagnose the difficulties that their students were having with mathematical word problems. The following section of this paper will explore how teachers responded to NEA as an aspect of the Counting On program.

Teacher Learning
The professional learning of teachers within the Counting On program had evolved by 2007 into a conference lasting two days and attended by one or two volunteer teachers from each school who would act as facilitators. The facilitators would then return to their school, form and train a team, and then implement the program. While initially a "train the trainer" model may have been a loosely accurate description of this process, it evolved and now a better term would be to call it a "facilitated model" of teacher professional learning. Whereas in a "train the trainer" process there would be an expectation of a consistency in delivery with everyone trained and then delivering the training in the same way, a facilitated model allows for more variability within regards to how the program is implemented because the expectation is that the facilitator will shape the program to meet local needs. This change meant that whereas cascade models of "train the trainer" type suffered from "dilution" as the process moved from level to level, in contrast a facilitated model had the potential to be both better and worse than the original facilitator training. The evaluation reports highlighted the success of the program depended to a great extent upon the school facilitator. Thus the implementation of NEA depended upon the school facilitator.

The concern with the teacher understanding of NEA levels mentioned in the earlier section was investigated in the 2007 Counting On evaluation report. It explored reasons for the regression of student outcome levels and listed factors such as the use of different teacher assessors, poor initial teacher understanding of the LFIN and NEA, misdiagnosis,
student resistance to assessment. It appeared that the majority of errors originated from the same small number of facilitators.

The 2007 evaluation reported the majority of teachers were strongly positive about the inclusion of NEA into the program. Many told of how it had been adapted across subjects and different stages. Teachers reported it being an understandable, easy to use, framework and process for uniting numeracy and literacy and there were requests for further opportunities for teacher professional learning involving NEA. It was observed that there was a divide between primary and secondary teachers. NEA appears to resonate more easily with primary teachers and with the issues of "numeracy across the curriculum" and "every teacher being a teacher of literacy". Primary teachers were able to use it to analyse Basic Skills Test errors (schools receive a report on their students who sat for the NSW state wide primary school testing program) and develop strategies to improve their students' literacy needs. In the secondary school, the resonance was not as high, resulting in some secondary teachers regarding NEA as an issue that was not their concern. The report stated a typical comment extract was 'The inclusion of NEA has been extremely beneficial in providing teachers with new insights into where and why the students break down in solving word number problems. The workshops we have provided have indicated that a number of secondary mathematics teachers find it difficult to embrace this process." However it should be emphasised that this does not represent all secondary teachers, as is evidenced by the following comment "One head teacher has adopted/adapted it to assist senior students in Stage 6 mathematics" (White, 2008, p. 12).

The 2008 evaluation report described how teachers had extended the use of NEA beyond a diagnostic tool to a pedagogical and remedial tool. Teachers displayed the prompts as a poster in the classroom (see Figure 2) and all students were expected to work through the NEA levels for all mathematical problems.

Figure 2. The classroom poster.

In a whole class setting, students worked aloud in order to scaffold the learning of those struggling with one or more of the levels. Teachers also used it as a problem solving approach, seen in this teacher response:

The Newman's error analysis and follow-up strategies have helped students with their problem-solving skills, and teachers have developed a much more consistent approach
to the teaching of problem-solving. Not only has it raised awareness of the language demands of problem solving, but through this systematic approach, teachers can focus on teaching for deeper understanding (White, 2009, p. 42).

Thus a teacher working with an individual student would ask the student to work through each prompt (see Figure 1) and the teacher would assist the student with difficulties. Thus reading and comprehension problems would be dealt with in the usual ways by the teacher. For example, generally primary students would be put on a remedial program whereas secondary students would be referred to the English department. Comprehension problems would often be remedied through the use of diagrams, drawings or concept maps. However, it is the transformation level or the "mathematizing" level that is often considered the most difficult to remedy by teachers.

![Figure 3. A tape diagram for the word problem in Figure 1.](image)

The NSW Department of Education Curriculum Support Directorate has developed teacher material involving the use of what are known as "tape diagrams" (see Figure 3 above) to assist teachers in supporting students who experience difficulties at this level. The tape diagram acts as an organising tool to assist the student keep track of all the data in the word problem. The last two levels of processing and encoding are the usual fare of teachers.

**Conclusion**

The Counting On program was a success in improving both teacher and student learning outcomes involving NEA. The teachers extended their initial understanding of NEA as a diagnostic procedure to a broader pedagogical understanding. The data revealed a statistically and educationally significant improvement existing in student learning outcomes between the start and the completion of the program involving mathematical problem solving using word problems. As well, NEA is being used by teachers as a remedial classroom strategy and as a wider classroom pedagogical and problem solving
strategy. Thus NEA assists teachers and students to appreciate the power of mathematics in making sense of their world, for

... if the essence of mathematics is the setting up of and working with mathematical models, and if we treat word problems in such a way, then they might have a role to play in helping children better understand the process of mathematizing. And with the increasing mathematizing of the world (from national test scores to pension prospects), informed and critical citizens need to be aware that mathematizing is not something that arises from the world, but something that is done to the world. In a small way, working on word problems might help begin to develop this awareness (Askew, 2003, p. 85).

So in summary, this paper concludes that NEA is a powerful diagnostic assessment and teaching tool.

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